

## **Geometry Instructional Focus Documents**

### Introduction:

The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

### Evidence of Learning Statements:

The evidence of learning statements are guidance to help teachers connect the Tennessee Mathematics Standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee Mathematics Standards. These statements are divided into four levels. These four levels are designed to help connect classroom assessments with the performance levels of our state assessment. The four levels of the state assessment are as follows:

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 4: Performance at this level demonstrates that the student has an extensive understanding and expert ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in this same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting.

### Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

## Congruence (G.CO)

### Standard G.CO.A.1 (Supporting Content)

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, plane, distance along a line, and distance around a circular arc.

#### Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify a point, line, line segment, ray, plane, angle, circle, and parallel and perpendicular lines from a simple two-dimensional figure.</p> <p>Differentiate between a line, line segment, and ray from a two-dimensional and three-dimensional figure.</p>	<p>Define an angle, circle, perpendicular line, parallel line, and line segment in simple terms.</p> <p>Identify a point, line, line segment, ray, plane, angle, circle, and parallel and perpendicular lines from two-dimensional composite figures and three-dimensional figures.</p>	<p>Generate a precise definition of an angle, circle, perpendicular line, parallel line and line segment based on the undefined notions of points, lines, planes, and the distance along a line and around an arc.</p>	<p>Compare and contrast different versions of precise definitions of an angle, circle, perpendicular line, parallel line, and line segment. Provide mathematical justification for which definition is the most precise.</p>

### Instructional Focus Statements

#### Level 3:

Students begin to develop an understanding of points, lines, and planes in elementary grades. In high school, students should extend this understanding in order to self-generate more precise definitions of angle, circle, perpendicular line, parallel line, and line segment. This should be cultivated through experiences with rigid motions that provide insights on how the terms might be defined rather than merely memorizing definitions. Students should be encouraged to develop their own definitions. Discussing student-generated definitions and challenging the evidence used to form the definitions will lead to a deeper conceptual understanding of the terms. Students should understand that a point, line, and plane are undefined terms with simplistic definitions. A point is a location and has no size. A line is a one-dimensional object that extends infinitely in either direction but has no width. A plane is a

two-dimensional object that extends infinitely with no edges or height. These three undefined terms are the cornerstone on which students define all other terms in Geometry.

As students understand the basic undefined terms, they should be allowed time to explore angles, circles, line segments, and parallel and perpendicular lines to develop their own precise definitions. In this exploration, students should compare images of what an object is and is not, eventually creating their own images and counterexamples. A Frayer model can be a helpful tool for students to organize their thinking to develop student-generated definitions. It is beneficial to do this after students have worked with transformations, since these definitions are derived from transformations. For example, a line parallel to another is the translation of the original line.

A common misconception with angles is that students often think of an angle in a polygon as the vertex (a point) instead of the relationship between the consecutive sides or the degrees of the opening. Students should be asked questions to guide them to see an angle as a ray rotated about its endpoint by a specific degree. It is imperative that students develop an in-depth conceptual understanding of geometric definitions through hands-on discovery learning.

**Level 4:**

Students at this level should be challenged to critiquing others' precise definitions providing counterexamples when appropriate, adding to the definition as needed, or correcting misconceptions evident in the definition. This can be achieved by students critiquing other students' definitions or by looking at teacher created definitions that were intentionally created to guide students to look for specific misconceptions.

**Standard G.CO.A.2 (Supporting Content)**

Represent transformations in the plane in multiple ways, including technology. Describe transformations as functions that take points in the plane (pre-image) as inputs and give other points (image) as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify when a figure has been rotated, reflected, or translated in a picture or on a graph.</p> <p>Identify when a figure has been dilated, stretched, or compressed in a picture or on a graph.</p>	<p>Generate a single rigid transformation on or off a coordinate plane.</p> <p>Generate a dilation of a simple figure on or off a coordinate plane.</p> <p>Determine equal distances and congruent angle measures in given figures.</p>	<p>Use multiple representations to represent transformations: in words, algebraically, graphically, and in a table of values.</p> <p>Use technology to perform a given transformation and identify the effects of transformation(s) on a figure.</p> <p>Describe transformations as functions in which points in the pre-image are input values that result in points in the image as output values.</p>	<p>Critique transformations drawn by others based on a given description, providing mathematical justification for their thinking.</p> <p>Critique others' description of a transformation represented in words, algebraically, graphically, or in a table of values, providing mathematical justification for their thinking.</p> <p>Represent a sequence of transformations that includes a combination of a rigid motion and a dilation in words, algebraically, graphically, and in a table of values.</p>

## **Instructional Focus Statements**

### **Level 3:**

Students begin exploring rigid motion in grade 8. In Algebra I, they identified the effects on the graphs of a function by replacing  $f(x)$  with  $f(x) + k$ ,  $kf(x)$ , and  $f(x + k)$  in standard A1.F.BF.B.2. In this course, they should connect these concepts by recognizing that  $f(x) + k$  is a vertical translation,  $f(x + k)$  is a horizontal translation, and  $kf(x)$  is a vertical dilation, sometimes informally referred to as a stretch or compression. This is also explored further in Algebra II standard A2.F.BF.B.3.

Students should be exposed to multiple ways to represent all transformations including words, coordinate notation, function notation, graphs, and technology. Students should also discover that rigid motions preserve distance and angle measures, while other transformations, such as dilations, do not. They should be able to distinguish between a transformation that is rigid motion and one that is not. Allowing students to explore transformations using dynamic Geometry software is beneficial in discovering these differences.

### **Level 4:**

As students extend their understanding of representing, describing, and comparing transformations, they should be able to analyze others' transformations or descriptions of transformations. This will help develop a deep understanding of the connections between the different representations. These samples need to include all representations of transformations including drawings, words, algebraic and graphical representations, and in a table of values.

Extending this standard to include a sequence of transformations will help students bridge the Algebra I standard A1.F. BF.B.2 to the Algebra II standard A2.F.BF.B.3, which requires them to work a larger set of function family problems involving transformations.

**Standard G.CO.A.3 (Supporting Content)**

Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry the shape onto itself.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Choose a rectangle, parallelogram, trapezoid, or regular polygon that indicates all possible lines of symmetry.</p> <p>Determine if a rectangle, parallelogram, trapezoid, or regular polygon has rotational symmetry.</p> <p>Draw a line of symmetry when given a rectangle, parallelogram, trapezoid, or regular polygon.</p>	<p>Explain symmetry in terms of transformations.</p> <p>Draw multiple lines of symmetry when given a rectangle, parallelogram, trapezoid, or regular polygon.</p> <p>Determine if a rectangle, parallelogram, trapezoid, or regular polygon, has rotational symmetry, limited to 90 or 180 degrees.</p>	<p>Determine a line of symmetry and/or the degree of rotational symmetry that exist in a rectangle, parallelogram, trapezoid, or regular polygon.</p> <p>Describe the rotations and/or reflections that carry a rectangle, parallelogram, trapezoid, or regular polygon onto itself.</p> <p>Determine the attributes of a figure based on its symmetries.</p>	<p>Create figures based on a given set of transformations that allows any two-dimensional figure to be carried onto itself.</p> <p>Synthesize and explain the relationships that exists between the symmetries of a figure and its geometric attributes.</p>

**Instructional Focus Statements**

**Level 3:**

Initially, instruction should focus on having students explore transformations with rectangles, parallelograms, trapezoids, and regular polygons enabling them to discover which ones cause the image to exactly overlap the pre-image. This often confuses students at first but is essential in their conceptual understanding of this standard. Students should be provided with appropriate tools to aide with their explorations such as tracing paper, mirrors, and dynamic Geometry software. These explorations should lead to students defining a set of transformations in which the image ends up exactly the matching the pre-image, ultimately leading students to recognize the mathematical connection between transformations and the symmetries within a figure. For example, while learning about rectangles, students should build on their understanding by connecting symmetry with the attributes of the

figure. Students should be able to compare the symmetries of a rectangle with quadrilaterals that are not rectangles to better understand the similarities and differences of each polygons attributes. This standard lays the groundwork for G.CO.A.4.

**Level 4:**

Students should have ample opportunities to compare figures that have and do not have symmetry. They should be encouraged to use these comparisons to explain the connection between the symmetries and the geometric attributes of a figure. To solidify this understanding, students should be able to create their own examples of figures with one or more types of symmetry. Additionally, students should be able to explain their reasoning with precise mathematical vocabulary.

**Standard G.CO.A.4 (Supporting Content)**

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify rotations, reflections, and translations in a picture or graph.	Describe rotations, reflections, and translations in general terms.  Measure and construct angles, line segments, and circles both on and off a coordinate plane.	Develop and give a student-generated definition of a rotation in terms of distances, angles, and arcs.  Develop and give a student-generated definition of a reflection in terms of distance, and parallel and perpendicular lines.  Develop and give a student-generated definition of a translation in terms of distance and parallel lines.  Generate precise definitions of rotations, reflections, and translations such that they are unique to the given transformation.	Critique others' definitions of rotations, reflections, and translations, paying particular attention to the level of precision and providing counter-examples when they exist.



## **Instructional Focus Statements**

### **Level 3:**

This standard is horizontally aligned with the other standards within the congruence cluster. One of the most difficult aspects of this standard for students is the level of precision required to generate definitions for rotations, reflections, and translations. Students should already have experience with transformations prior to independently developing these definitions. They should also have experience with appropriate tools such as protractors, compasses, and/or dynamic geometry software allowing them to precisely measure the angles and construct the arcs which occur during the transformations. It is important that students be allowed adequate time to explore transformations so that they have the appropriate conceptual understanding that allows them to develop the definitions on their own. Students may be encouraged to use the properties of each transformation and focus on the relationships that exist between the lines and angles in the pre-image and the lines and angles in the original image. Students should be provided examples of transformed figures and asked to explain why or why not a relationship can be described as a specific rigid motion. It is vital that students attend to precision when writing their definitions to ensure they are unique to the given transformation.

### **Level 4:**

Critiquing others' definitions will help students discover the necessary level of precision needed to accurately define these transformations. Providing students with both examples and non-examples of rigid motions is vital to this process. Likewise, students who can self-generate a counter-example demonstrate a much deeper understanding of the properties of these transformations. As students create their own student-developed definitions and critique others' definitions, they should use examples, counter-examples, and precise mathematical vocabulary.

**Standard G.CO.A.5 (Supporting Content)**

Given a geometric figure and a rigid motion, draw the image of the figure in multiple ways, including technology. Specify a sequence of rigid motions that will carry a given figure onto another.

**Scope and Clarifications:**

Rigid motions include rotations, reflections, and translations.

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Choose the correct term to describe the rigid transformation from a picture.</p>	<p>Sketch the image of the figure, given a geometric figure and a translation described in words.</p> <p>Determine a single rigid motion that carries a given figure onto another.</p>	<p>Draw the image in multiple ways, including by hand, on or off a coordinate plane, or by using technology such as dynamic geometry software, given a geometric figure and a rigid motion (rotation, reflection, or translation).</p> <p>Draw the image in multiple ways, given a geometric figure and a sequence of rigid motions,</p> <p>Describe a sequence of rigid motions that will carry a given figure onto another.</p> <p>Recognize and explain that there can be more than one correct sequence that will map a given pre-image onto an image.</p>	<p>Describe multiple sequences that will map one to the other, given a pre-image and image.</p> <p>Critique others' drawings or descriptions, recognizing mistakes and correcting them by explaining their justifications using precise mathematical language.</p> <p>Compare the relationship between a sequence of transformations and a single transformation and understand that one transformation can more efficiently result in the same image. For example, a sequence of two reflections across parallel lines results in a translation.</p>

## **Instructional Focus Statements**

### **Level 3:**

Students began exploring rigid motions in grade 8 both on and off the coordinate plane. In this course, students should explore rigid motions further with appropriate tools to help them visualize each transformation including tracing paper, mirrors, and dynamic geometry software. They should generate transformations both on and off a coordinate plane. When working on a coordinate plane, students should explore the effects on the coordinates and begin to see transformations as functions with input values and output values. Instructions for this standard integrates nicely with standard G.CO.A.2. Once students have had ample experience exploring each transformation, students need to explore what happens when a sequence of transformations are applied. This will lead to students being able to determine the sequence applied to a given pre-image and image. This standard is the foundation to all the rigid motion standards in this cluster.

### **Level 4:**

Students should continue with their exploration of the effects on a figure when a sequence of rigid motions are applied. Finding alternate lists of sequences that will map a pre-image onto an image and recognizing that the order of the transformations may change the result will deepen their understanding of their properties. Additionally, students will solidify their understanding by critiquing others' applications of a specified sequence or description of a sequence provided a pre-image and image, analyzing their mistakes, if any, and providing corrections as needed.

**Standard G.CO.B.6 (Major Work of the Grade)**

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to determine informally if they are congruent.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define rigid motion.</p> <p>Choose the correct term to describe the rigid transformation given a picture.</p>	<p>Measure distances and angles of figures.</p> <p>Understand that congruent polygons have corresponding side lengths and angle measures that are equal or circles have radii that are equal in length.</p>	<p>Use definitions of rigid motions to perform a given transformation.</p> <p>Predict the effect of a given rigid motion on a figure.</p> <p>Determine what rigid motion(s) will map one figure on to another.</p> <p>Verify that two figures are congruent by measuring lengths and angle measures to ensure they are equal, given the pre-image and its image after a rigid motion,</p> <p>Make the connection that two figures are congruent because one is the resulting image of a rigid motion on the other.</p>	<p>Construct a viable argument on why a rigid motion results in congruent figures using the definition of rigid motions and the definition of congruence.</p> <p>Critique others' predictions of the effect of a given transformation on a figure.</p>

## **Instructional Focus Statements**

### **Level 3:**

In standard G.CO.A.4, students developed definitions of rigid motions. In this standard, students should be able to use those definitions to create the transformation. It is important that they perform the transformation on individual points in the figure rather than on the whole figure at once. For example, to perform a reflection of a given polygon in the y-axis, students need to draw a line perpendicular to the y-axis that passes through a vertex of the polygon and draw the reflection of the vertex at the same distance along that line on the other side of the y-axis.

Students also need to recognize that rigid motion results in a congruent figure. Therefore, two figures can be determined congruent if a rigid motion exists that maps one figure onto the other. This will require students to have already been exposed to standard G.CO.A.5. It is best for students to discover this concept by comparing side and angle measurements of figures that have been transformed by a rigid motion with those that have not. Students must practice precision in their measurements to verify these results.

This standard develops the understanding of the concept of using rigid motion to prove congruent figures which leads to other standards in this cluster where students will be using this skill.

### **Level 4:**

Students will deepen their understanding of this standard by describing why the image of a rigid motion results in a figure congruent to its pre-image. Providing students with examples of transformations that are and are not rigid motions and having them explain why the figures are congruent or not using the definitions will help students build their arguments. These examples can come from other student work or teacher created examples.

**Standard G.CO.B.7 (Major Work of the Grade)**

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

<b>Students with a level 1 understanding of this standard will most likely be able to:</b>	<b>Students with a level 2 understanding of this standard will most likely be able to:</b>	<b>Students with a level 3 understanding of this standard will most likely be able to:</b>	<b>Students with a level 4 understanding of this standard will most likely be able to:</b>
<p>Define rigid motion.</p> <p>Define congruence.</p> <p>Choose the correct term to describe the rigid transformation of one triangle to another from a picture.</p>	<p>Identify corresponding pairs of sides and angles given two figures that are marked or a congruence statement describing two figures.</p> <p>Perform a rigid motion of a given triangle both on a blank space and on a coordinate plane using appropriate tools such as a mirror, tracing paper, or dynamic geometry software.</p> <p>Find the measures of sides and angles of a triangle both on a blank space and on a coordinate plane using appropriate tools such as a ruler, protractor, distance formula, or dynamic geometry software.</p> <p>Understand that congruent figures have equal lengths and angle</p>	<p>Identify and name corresponding parts of triangles.</p> <p>Make the connection that two triangles are congruent because one is the resulting image of a rigid motion on the other.</p> <p>Verify that corresponding sides have equal lengths and corresponding angles have equal measures, given congruent triangles,</p> <p>Explain why the triangles are congruent using the definition of congruence in terms of rigid motion, given that the corresponding sides and angles of two triangles are congruent.</p> <p>Read and use correct notation that</p>	<p>Explore corresponding parts of other shapes to determine if the figures are congruent.</p> <p>Create examples of non-congruent triangles that have some congruent corresponding parts and explain why they are not congruent using the definition of congruence in terms of rigid motion.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	measurements.	<p>shows corresponding parts are congruent and triangles are congruent.</p> <p>Write a triangle congruence statement using correct notation.</p>	

### Instructional Focus Statements

#### Level 3:

This standard extends G.CO.B.6 to verify that in order for two figures to be congruent, their corresponding parts must also be congruent. This concept is not a difficult one, but the converse is often more challenging. If only some of the corresponding parts of two triangles are congruent does that mean that the triangles are congruent? Students can explore examples of when this is and is not the case.

It is important that students also know how to read and correctly notate congruent triangles (ex.:  $\triangle ABC \cong \triangle DEF$ ) and their corresponding parts. They must attend to precision when setting up these congruence statements and understand that the statement itself will clearly indicate the congruent corresponding parts when written correctly.

This discovery will lead nicely into standard G.CO.B.8 in which students will learn that, with triangles, they only need to know specific groups of congruent corresponding parts to know that all corresponding parts are congruent and this the triangles are congruent.

#### Level 4:

Allowing students to explore shapes other than triangles will help them understand that this concept extends to all figures. Students must use their critical thinking skills to create their own examples of triangles that have some congruent corresponding parts but do not result in congruent triangles. To extend this further, students need to use their communication skills to explain why this happens using the definition of congruence in terms of rigid motion.

**Standard G.CO.B.8 (Major Work of the Grade)**

Explain how the criteria for triangle congruence (ASA, SAS, AAS, and SSS) follow from the definition of congruence in terms of rigid motions.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify corresponding parts in two triangles.</p> <p>Write the congruence statement for each corresponding part (ie.: <math>\angle A \cong \angle D</math>, <math>AB \cong DE</math>)</p>	<p>Determine if two triangles are congruent in terms of rigid motion using appropriate tools such as mirrors or tracing paper</p> <p>Determine if two triangles are congruent in terms of congruent corresponding parts using tools such as rulers, protractors, distance formula, etc.</p>	<p>Determine which combinations of congruent corresponding parts must be known to verify that two triangles are congruent.</p> <p>Explain how knowing SSS, SAS, ASA, or AAS is enough to say that two triangles are congruent using the definition of congruence in terms of rigid motions.</p> <p>Read and use correct notation that shows corresponding parts are congruent and triangles are congruent.</p> <p>Provide instructions to another student giving only three measurements of triangle sides and/or angles so that they can accurately draw a triangle congruent to their own drawing.</p>	<p>Explain and provide examples of why two combinations of three congruent corresponding parts (AAA and SAA) do not prove triangles congruent.</p> <p>Critique the work of others in determining congruent triangles based on SSS, SAS, ASA, or AAS and explain why that work is correct or incorrect.</p> <p>Explain the effects on these properties when the triangles are right triangles.</p>



## **Instructional Focus Statements**

### **Level 3:**

This standard builds on G.CO.B.7, where students have to verify that all corresponding parts of two triangles must be congruent to say the triangles are congruent. In this standard, students explore the properties of triangles to lead to a discovery that if certain groups of corresponding parts in two triangles are congruent, then the rest of them will be, too. Therefore, students only need to determine specific criteria to prove that two triangles are congruent (ASA, SAS, AAS, and SSS).

Instruction should focus on allowing students to explore combinations of congruent sides and/or angles to determine which combinations work and which ones do not work. Physical manipulatives allow students to create triangles based on given congruent parts and then compare them to determine if they are congruent. For example, a pair of students can each cut three straws to given lengths, assemble them into a triangle, and then hold the triangles next to each other to see if they are congruent by SSS. Students can also measure the angle between two straws and tape them to hold that angle measure along with other given information to compare triangles using SAS, AAS, or ASA. Guide students to see that by holding the triangles they are comparing next to each other, they are physically performing the rigid motion. This will help them see that the congruence criteria (ASA, SAS, AAS, and SSS) can be verified by the definition of congruence in terms of rigid motion. Dynamic geometry software is also a useful tool to help students see how this works.

Students often struggle to understand the difference between ASA and AAS or SAS and SSA (which does not work). Guide students to see that ASA and SAS indicate the included corresponding side or angle must be congruent, whereas AAS and SSA indicate a non-included side or angle. Allowing students to explore the combinations using physical manipulatives or dynamic geometry software can help. To solidify their understanding, challenge students to draw a triangle on paper and then give a partner a combination of 3 side lengths and/or angle measures such that they can draw a triangle that is congruent to their own. (Ex. Draw a 3 inch segment with a 40 degree angle on one end and a 60 degree angle on the other end.)

### **Level 4:**

Challenge students to demonstrate using precise drawings or dynamic geometry software examples that show why two combinations of congruent corresponding parts (AAA and SAA) do not prove triangles congruent. Provide examples of others' work for students to critique. This can be student work or teacher created examples. Some should be correct and some incorrect. Have students explain why the conclusion is correct or incorrect and verify using the definition of congruence in terms of rigid motions. Have students further explore this standard using right triangles. Ask them to determine if the pre-determined combinations still work and if the combinations that don't work for non-right triangles work for right triangles, leading them to discover the combination SAA actually works if and only if it is a right triangle (HL).

**Standard G.CO.C.9 (Major Work of the Grade)**

Prove theorems about lines and angles

**Scope and Clarifications:**

Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs. Theorems include but are not limited to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify special lines, such as parallel, perpendicular, and bisectors.</p> <p>Identify special pairs of angles, such as supplementary, complementary, vertical, and those formed by lines crossed by a transversal.</p>	<p>Understand the relationships of lines, such as perpendicular bisectors.</p> <p>Understand the relationships of special pairs of angles, such as complementary angles have a sum of 90 degrees.</p> <p>Recognize pairs of lines or angles as a rigid motion that results in congruence.</p> <p>Give an informal explanation of the relationship of pairs of lines and/or angles.</p> <p>Complete a partial proof by filling in the blanks when given either a statement or a reason.</p>	<p>Make conjectures about the relationships between lines and/or angles.</p> <p>Prove those conjectures are true using precise mathematical language and a logical order of statements.</p> <p>Use rigid motions to prove the relationship between the figures in the conjectures.</p> <p>Construct a two-column proof or paragraph proof.</p> <p>Compare their proof with other students' proofs or teacher created proof examples.</p>	<p>Analyze and critique proofs written by others by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument.</p> <p>Improve their own proofs based on what they have seen from others' proofs.</p>

## **Instructional Focus Statements**

### **Level 3:**

Proofs, both formal and informal, should be addressed multiple times throughout this course. Constructing a proof is challenging and requires practice. Requiring students to informally explain properties and relationships of lines and angles to justify answers to problems will lead students into constructing more formal two-column and paragraph proofs. Having students make conjectures about those relationships will help students recognize the need for the proof and learn to think more strategically. Instruction should focus on using precise language and a logical order for their reasoning. It is particularly helpful to allow students to talk through their reasoning with another student to verify their language and logical order before writing it down.

Depending on the selected approach to the proof, students should be encouraged to draw on their experience with transformations and congruence as well as other prior knowledge to prove conjectures about relationships of lines and angles.

Students need to be familiar with different types of proofs including two-column proofs and paragraph proofs. Having students fill in a partially completed proof to start with can be helpful before creating proofs on their own. Comparing their proofs with others can help them better understand the expectations of the reasoning, order, and precise language required in a formal proof.

### **Level 4:**

Students need to be exposed to other proofs to analyze and critique. These should be a combination of accurate proofs and some with mistakes. Students should provide counterexamples with their explanation when disproving these proofs. These critiques will help students develop a deeper understanding of the logic of proofs and the need for clear and precise language that they can then apply to their own proofs.

**Standard G.CO.C.10 (Major Work of the Grade)**

Prove theorems about triangles.

**Scope and Clarifications:**

Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs. Theorems include but are not limited to: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define congruent. Identify congruent figures in a picture using symbols or given notation.</p> <p>Define complementary and supplementary angles.</p> <p>Define parallel lines. Identify parallel lines in a picture using symbols or given notation.</p> <p>Identify the angle opposite a specified side in a triangle and the side opposite a specified angle in a triangle.</p>	<p>Know that the sum of the angles in a triangle is 180 degrees.</p> <p>Write a statement relating two figures using correct mathematical notation. Such as <math>\angle A \cong \angle B</math> or <math>\overline{CD} \parallel \overline{EF}</math></p> <p>Define congruence in terms of rigid motion. Recognize when rigid motions can be used to prove congruent corresponding parts in a given figure.</p>	<p>Define median of a triangle, identify or draw it in a picture.</p> <p>Explore the properties of triangles and make conjectures.</p> <p>Formally prove the conjectures using precise mathematical language.</p> <p>Use rigid motions to prove the conjectures.</p> <p>Construct a two-column proof or paragraph proof.</p> <p>Compare their proof with other students' proofs or teacher created proof examples.</p>	<p>Analyze and critique proofs written by others by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument.</p> <p>Collaborate and critique proofs, given proofs written by others. .</p>

## **Instructional Focus Statements**

### **Level 3:**

This standard is an extension to G.CO.C.9, since those skills will be required for this standard as well. Students should be allowed to explore and make their own conjectures about triangles and their properties using appropriate tools such as a compass, protractor, tracing paper, or dynamic geometry software.

Note that this is the first time students will be introduced to a median of a triangle. This is a great place to review constructions and allow students to use appropriate tools to construct the medians in a triangle. Students should be encouraged to start with different types of triangles (acute, obtuse, right, etc.), attend to precision in their construction, and make observations to lead them to the conjecture that the medians will always meet at a single point.

Proofs, both formal and informal, should be addressed multiple times throughout this course. Constructing a proof is challenging and requires practice. Requiring students to informally explain the properties of triangles will lead students into constructing more formal two-column and paragraph proofs. Having students make conjectures about those relationships will help students recognize the need for the proof and learn to think more strategically. Instruction should focus on using precise language and a logical order for their reasoning. It is particularly helpful to allow students to talk through their reasoning with another student to verify their language and logical order before writing it down. Classroom discussions can help students broaden their thinking by making them consider different approaches to a proof. It will also help students develop more precise mathematical language. Having students restate other student's informal explanations in a more formal and precise way will help them focus on the language. Having students find and correct mistakes in an incorrect proof will help them better understand the organization and flow of a proof.

Students need to apply their prior knowledge to their proofs. Therefore, depending on the approach taken, students may be encouraged to draw on their experience with transformations, symmetry, and congruence to prove conjectures about triangles and their properties.

Students need to be familiar with different types of proofs including two-column proofs and paragraph proofs. Having students fill in a partially completed proof to start with can be helpful before creating proofs on their own. Comparing their proofs with others can help them better understand the expectations of the reasoning, order, and precise language required in a formal proof.

### **Level 4:**

Students need to be exposed to other's proofs to analyze and critique. These should be a combination of accurate proofs and some with mistakes. Students should provide counterexamples with their explanation when disproving these proofs. These critiques will help students develop a deeper understanding of the logic of proofs and the need for clear and precise language that they can then apply to their own proofs.

**Standard G.CO.C.11 (Major Work of the Grade)**

Prove theorems about parallelograms.

**Scope and Clarifications:**

Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs. Theorems include but are not limited to: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify opposite and consecutive sides or angles in a figure.</p> <p>Define congruent. Identify congruent figures in a picture using symbols or given notation.</p> <p>Define parallel lines. Identify parallel lines in a picture using symbols or given notation.</p> <p>Define perpendicular. Identify perpendicular lines in a picture using symbols or given notation.</p> <p>Define supplementary angles.</p> <p>Define diagonals of a polygon and identify or draw them in a given figure.</p>	<p>Define a parallelogram as a quadrilateral with two pairs of opposite parallel sides.</p> <p>Write a statement relating two figures using correct mathematical notation. Such as <math>\angle A \cong \angle B</math> or <math>\overline{CD} \parallel \overline{EF}</math></p> <p>Define congruence in terms of rigid motion. Recognize when rigid motions can be used to prove congruent corresponding parts in a given figure.</p> <p>Prove two triangles are congruent.</p>	<p>Explore the relationships that exist between the sides, angles, and diagonals of parallelograms, including rectangles, rhombuses, and squares.</p> <p>Make conjectures about the properties of parallelograms, rectangles, rhombuses, and squares.</p> <p>Formally prove the properties of parallelograms using precise mathematical language.</p> <p>Use rigid motions to prove the conjectures.</p> <p>Construct a two-column proof or paragraph proof.</p>	<p>Analyze and critique proofs written by others by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument.</p> <p>Improve their own proofs based on what they have seen from others' proofs.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define bisect in terms of a line segment or angle.		Compare their proof with other students' proofs or teacher created proof examples. Explain the relationships between parallelograms, rectangles, rhombuses, and squares.	

### Instructional Focus Statements

#### Level 3:

This standard is an extension to G.CO.C.9 and G.CO.C.10, since those skills will be applied here as well. Students should be allowed to explore and make their own conjectures about the relationships between the sides, angles, and diagonals in a parallelogram using appropriate tools such as a compass, protractor, tracing paper, or dynamic geometry software.

Proofs, both formal and informal, should be addressed multiple times throughout this course. Constructing a proof is challenging and requires practice. Requiring students to informally explain properties and relationships of lines and angles in parallelograms, rectangles, rhombuses, and squares will lead students into constructing more formal two-column and paragraph proofs. Having students make conjectures about those relationships will help students recognize the need for the proof and learn to think more strategically. Instruction should focus on using precise language and a logical order for their reasoning. It is particularly helpful to allow students to talk through their reasoning with another student to verify their language and logical order before writing it down. Classroom discussions can help students broaden their thinking by realizing how they are limiting themselves. It will also help students develop more precise mathematical language. By having students restate other student's informal explanations in a more formal and precise way, will help them focus on the language.

Students need to apply their prior knowledge to their proofs. Therefore, depending on the approach taken, students may be encouraged to draw on their experience with transformations and congruence to prove conjectures about relationships of lines and angles.

Students need to be familiar with different types of proofs including two-column proofs and paragraph proofs. Having students fill in a partially completed proof to start with can be helpful before creating proofs on their own. Comparing their proofs with others can help them better understand the expectations of the reasoning, order, and precise language required in a formal proof.

This standard should also lead to a comparison of the properties of a parallelogram, rectangle, rhombus, and square to help students develop an understanding of the relationship between these figures. For example, a rectangle is a parallelogram because it holds all the same properties of a parallelogram, but is special because it also has four congruent angles. Likewise, a square holds all the properties of a rectangle, but it is even more special because it also holds all the properties of a rhombus.

**Level 4:**

Students need to be exposed to other's proofs to analyze and critique. These should be a combination of accurate proofs and some with mistakes. Students should provide counterexamples with their explanation when disproving these proofs. These critiques will help students develop a deeper understanding of the logic of proofs and the need for clear and precise language that they can then apply to their own proofs.



**Standard G.CO.D.12 (Supporting Content)**

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

**Scope and Clarifications:**

Constructions include but are not limited to: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; constructing a line parallel to a given line through a point not on the line, and constructing the following objects inscribed in a circle: an equilateral triangle, square, and a regular hexagon.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define congruency and recognize symbols that indicate congruent line segments or symbols in a picture.</p> <p>Define bisect and recognize symbols that indicate a bisected line segment or angle in a picture.</p> <p>Define parallel and perpendicular and recognize symbols that indicate parallel and perpendicular lines in a picture.</p>	<p>Understand that copying a segment or angle means creating an exact replica that is congruent to the original.</p> <p>Understand that a bisector lies exactly in the middle of a figure.</p> <p>Use a compass to construct circles and curves.</p> <p>Define a circle, equilateral triangle, square, and regular hexagon.</p>	<p>Develop methods using a variety of appropriate tools (compass, straightedge, string, reflective device, paper folding, etc.) to precisely construct geometric objects such as a perpendicular bisector and parallel lines.</p> <p>Use the virtual compass and line tool in dynamic geometry software to construct various geometric objects.</p> <p>Construct an equilateral triangle, square, and regular hexagon in a circle using appropriate tools such as a compass, straightedge, paper folding, graph paper, etc.</p> <p>Explain informally why and how</p>	<p>Compare various constructions to determine similarities and differences (ie. bisecting a segment and bisecting an angle).</p> <p>Write an informal or formal proof using precise math vocabulary why these construction methods work using the geometric relationships between the objects in the construction.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		these construction methods work. Understand the importance of precision in these constructions. Attend to precision when performing geometric constructions.	

### Instructional Focus Statements

#### **Level 3:**

Students must be allowed to develop a method to make these constructions rather than be given specific instructions to follow. They will need a basic understanding of the expected outcome. However, it is through the process of the construction and particularly developing the method that students will develop a deeper understanding of the properties of these objects. Students will want to use a ruler to bisect a line segment or a protractor to bisect an angle, but when performing these formal constructions, students should not use tools that measure. Instead they need to focus on the properties of the figures in the construction. Likewise, when students are using the dynamic geometry software, they should avoid using the automatic commands for bisecting or copying and use the virtual compass and line tool instead.

Precision is essential in performing these constructions or they will not work. For example, a perpendicular bisector construction may not end up exactly in the middle or perpendicular. Dynamic geometry software may help students perform the constructions precisely, particularly for students who struggle with using the tools precisely, but they need to also experience other methods. Developing the process of the methods is equally important for developing the understanding of the properties. Therefore, it is important that students be required to show their understanding by informally explaining what the methods used does and why it works.

#### **Level 4:**

The different constructions have some similarities. Allowing students to compare the methods for the different constructions to find those similarities will help them develop a deeper understanding of what makes them work. It will also help students better realize the importance of precision. This will in turn help students develop a more formal proof of the fact that each method works.

## SIMILARITY, RIGHT TRIANGLES, and TRIGONOMETRY (G.SRT)

### Standard G.SRT.A.1 (Major Work of the Grade)

Verify informally the properties of dilations given by a center and a scale factor.

#### Scope and Clarification:

Properties include but are not limited to: a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center of the dilation unchanged; the dilation of a line segment is longer or shorter in the ratio given by the scale factor.

#### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Distinguish between a dilation and a translation, reflection, or rotation.  Choose a dilation given a center and a scale factor.	Perform a dilation in the coordinate plane centered at the origin.  Choose properties of a dilation given by a center and a scale factor.  Choose a scale factor given a visual representation of a dilation and a center.	Determine the properties of a dilation given by a center and a scale factor.  Perform a dilation in the coordinate plane given a scale factor and a center.	Illustrate a dilation and explain the properties between the images using precise mathematical language, given a pre-image, center, and scale factor.

#### Instructional Focus Statements

#### Level 3:

In grade 8, students are introduced to the foundation of dilations. They describe the effect of dilations on two-dimensional figures using coordinates. In high school geometry, students should extend their understanding of dilations by identifying and verifying properties that are formed between a pre-image and a dilated image. Students should use precise mathematical justification to explain that dilations preserve angle measure, betweenness, and collinearity.

Students should also be able to understand and demonstrate that dilations map a line segment (the pre-image) to another one segment whose length is the product of the scale factor and the length of the pre-image. Additionally, students should be able to understand and demonstrate that dilations map a line not passing through the center of dilation to a parallel line and leave a line passing through the center unchanged.

As students solidify their understanding of dilations they should be using a range of scale factors, including scale factors less and greater than 1 to gain a full understanding of how dilations can shrink as well as expand figures.

#### **Level 4:**

As students verify informally the properties of dilations, they should be able to work with a range of dilations where the scale factor is greater and less than 1 and a variety of centers of dilation. Students should be able to illustrate dilations using a variety of tools including but not limited to paper and pencil, dynamic geometry software, and the coordinate plane.

As students make connections to relate the sides of the image and pre-image of a dilation, they should be able to justify their reasoning using precise mathematical vocabulary.

**Standard G.SRT.A.2 (Major Work of the Grade)**

Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

**Scope and Clarification:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Distinguish between a dilation and a translation, reflection, or rotation.</p> <p>Choose a dilation given a center and a scale factor.</p> <p>Choose two congruent triangles given all corresponding congruent angles and sides.</p> <p>Choose corresponding congruent sides and angles, given a triangle congruence statement.</p>	<p>Perform a dilation in the coordinate plane centered at the origin.</p> <p>Choose properties of a dilation given by a center and a scale factor.</p> <p>Choose a scale factor given a visual representation of a dilation and a center.</p> <p>Choose two similar triangles given all congruent corresponding angles and the proportionality of all corresponding pairs of sides.</p> <p>Choose congruent corresponding angles and the proportionality or all corresponding pairs of sides when given a triangle similarity statement</p> <p>Identify that two triangles are similar given that the triangles have</p>	<p>Determine the properties of a dilation given by a center and a scale factor.</p> <p>Perform a dilation in the coordinate plane given a scale factor and a center.</p> <p>Determine if two figures are similar using the definition of similarity in terms of similarity transformations.</p> <p>Identify which transformations preserve similarity for two triangles.</p> <p>Use similarity transformations to verify that all corresponding pairs of angles are congruent and verify the proportionality or all corresponding pairs of sides to show that the triangles are similar when given two triangles.</p>	<p>Illustrate a dilation and explain the properties between the images using precise mathematical language when given a pre-image, center, and scale factor.</p> <p>Explain the relationship between similarity transformations and the similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</p> <p>Explain the definition of similarity both in terms of transformations and in terms of measurements of corresponding sides and angles.</p> <p>Create a pair of similar figures with a sequence of dilations and rigid motions and mathematically justify that the figures are similar.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	<p>been transformed by a single rigid transformation followed by a dilation.</p> <p>Choose a scale factor given a simple dilation.</p>	<p>Determine the sequence of transformations that have occurred in order to determine when two figures are similar.</p> <p>Identify the ratio of proportionality for similar figures.</p>	

### **Instructional Focus Statements**

#### **Level 3:**

In grade 8, students began to develop a conceptual understanding of the effects of transformations on two-dimensional figures and use informal arguments to establish facts about angles. In the high school geometry standards, students use their prior understandings of transformations to discover the definition of similarity and further to discover which transformations preserve similarity in figures. In doing so, students should be given ample opportunity to explore sequences of transformations to visually conceptualize the relationship between the sides and the angles of similar figures. This should be done using tools that include but are not limited to patty paper, graph paper, and technology. Through this exploration, students should discover the definition of similarity in terms of similarity transformations and understand that similar figures have congruent pairs of corresponding angles and all pairs of corresponding sides have the same constant of proportionality. As students solidify their understanding of similar figures, they should understand that the scale factor increases or decreases the lengths proportionally while the angles remain congruent.

#### **Level 4:**

As students extend their understanding, they should be able work with a variety of sequences of dilations and rigid motions that determine if the image and pre-image are similar. Students should be able to explain the definition of similarity both in terms of transformations and in terms of measurements of corresponding sides and angles and explain the relationship using precise mathematical symbols, illustrations, and language. Additionally, students should be able to create a sequence of dilations and rigid motions to demonstrate a pair of similar figures and justify the similarity and the relationship that exists between the corresponding sides and angles.

**Standard G.SRT.A.3 (Major Work of the Grade)**

Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

**Scope and Clarification:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify that two triangles are similar, given that two pairs of angles are congruent.</p> <p>Determine the third angle, given two angles in a triangle.</p>	<p>Determine if they are similar using the AA criterion, given two pairs of independently drawn triangles.</p>	<p>Determine if two triangles are similar or not similar by AA criterion using properties of similarity transformations.</p>	<p>Derive the AA criterion for similarity of triangles and explain the justification using precise mathematical vocabulary.</p> <p>Determine if two triangles are similar or not similar by AA, SSS, and SAS criteria, using properties of similarity transformations.</p> <p>Create similar triangles and prove that they are similar using the AA criterion using precise mathematical justification and vocabulary.</p>

**Instructional Focus Statements**

**Level 3:**

Students began to develop a conceptual understanding of similar triangles in grade 8. In standard G.SRT.A.2, students gain an understanding that by using similarity transformations you can determine if two triangles are similar. Students grasped an understanding that the meaning of similarity for triangles is that all corresponding pairs of angles are congruent and all corresponding pairs of sides have the same constant of proportionality. This understanding,

coupled with the angle sum theorem learned in the middle grades, results in students recognizing when pairs of corresponding angles are congruent, then the triangles are similar without verifying the proportionality of the side lengths, culminating in establishing the AA criterion.

**Level 4:**

As student gain an in-depth understanding of using the properties of similarity transformations to establish the AA similarity criterion for triangles, including SSS (side-side-side) and SAS (side-angle-side), they should be able to generalize and explain what measurements are needed to ensure that two triangles are similar. Students should also be able to provide illustrations with mathematical justifications, explaining which criteria is appropriate to prove similarity in which situations. Additionally, while the standards only explicitly states the AA similarity criterion, students should also discover other similarity criteria that work such as SSS (side-side-side) and SAS (side-angle-side), where the lengths of the corresponding pairs of sides are proportional.



**Standard G.SRT.B.4 (Major Work of the Grade)**

Prove theorems about similar triangles.

**Scope and Clarification:**

Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs. Theorems include but are not limited to: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Complete a proportionality statement by choosing a missing side length, given two similar triangles that are formed by a line parallel to one side of the triangle.</p> <p>Choose the missing reason(s) that completes the proof, given a partial two-column proof.</p>	<p>Choose a proportionality statement, given two similar triangles that are formed by a line parallel to one side of the triangle.</p> <p>Choose the statement(s) and/or reason(s) that completes the proof, given a partial two-column proof.</p> <p>Choose the order in which the statements should appear, given a deconstructed paragraph proof.</p>	<p>Prove theorems about similar triangles by completing two-column and paragraph proofs.</p> <p>Use triangle similarity to prove the Pythagorean Theorem and its converse.</p>	<p>Determine and fix the error and explain the reasoning using precise mathematical justification, given a two-column and paragraph proof with errors.</p> <p>Prove theorems about similar triangles to discover that the geometric mean is the length of two parts of a right triangle when a perpendicular is dropped from the right angle to the hypotenuse.</p>

**Instructional Focus Statements**

**Level 3:**

In this standard, students will apply their knowledge of similarity to prove various theorems about triangles. Students should have opportunities to explore situations and form conjectures that they can prove. In doing so, students should discover theorems, such as a line parallel to one side of a triangle divides the other two sides proportionally. Students should recognize that the line cuts the two sides proportionally and make the connection that the parallel lines form congruent corresponding angles and by the AA criteria the triangles are similar resulting in congruent angles and proportional side

lengths. Also, students should use triangle similarity to prove the Pythagorean Theorem and its converse. These are explicitly stated in the scope and clarification of the standard but are not an exhaustive list.

**Level 4:**

As students extend their understanding of proving theorems about similar triangles, they should be able to work with increasingly challenging proofs and explain their reasoning in written and verbal formats. The standard's scope and clarification explicitly gives a few examples of theorems to prove but is not an exhaustive list. An additional example could include proving theorems about similar triangles to discover that the geometric mean is the length of two parts of a right triangle when a perpendicular is dropped from the right angle to the hypotenuse. To solidify understanding, students should be able to explain their reasoning by using appropriate mathematical symbols, vocabulary, and justifications.

**Standard G.SRT.B.5 (Major Work of the Grade)**

Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures.

**Scope and Clarification:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose transformation relationships in simple geometric figures in cases where an image is not provided.	Choose transformation relationships in simple geometric figures in cases where an image is not provided.	Use transformations to determine relationships among simple geometric figures and to solve problems.	Apply congruence or similarity criteria to solve complex problems involving multiple concepts, and explain the geometric reasoning involved.

**Instructional Focus Statements**

**Level 3:**

Students expand their learning with proofs to a broader set of situations that may include triangle similarity or triangle congruence. Students should be able to work with a broad range of situations to provide arguments about their observations using triangle similarity and congruence to explain geometric relationships. As students form connections between pairs of similar and congruent triangles they should refer back to geometric transformations to explain how the triangles are related and establish the correspondence between the triangles. Students should also make prior course work connections and use triangle congruency criteria to determine properties of parallelograms and special parallelograms.

**Level 4:**

As students solidify their understanding of congruence and similarity criteria for triangles and how they are used to solve problems, they should extend their understanding by determining properties of parallelograms, special parallelograms, and other geometric figures. Students should be able to provide mathematical justifications explicitly linking similarity and congruence of triangles to other geometric figures. Additionally, students should be able to examine and critique justifications presented by others. This will help them develop more precise arguments of their own when solving real world problems.

**Standard G.SRT.C.6 (Major Work of the Grade)**

Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

**Scope and Clarification:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify the side in relationship to a given angle in a right triangle.</p> <p>Identify the angle in relationship to a given side in a right triangle.</p>	<p>Know the definition of trigonometric ratios using right triangles.</p> <p>Choose the missing sides and angles of a right triangle, given other sides and angles.</p>	<p>Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</p> <p>Find missing sides and angles of a right triangle, given other sides and angles.</p>	<p>Explain the relationship that exists between the ratios of the sides of right triangles that are proportional.</p> <p>Create right triangles that have and do not have proportional side lengths and explain the reasoning with precise mathematical justifications.</p>

**Instructional Focus Statements**

**Level 3:**

Students should be able to understand that by similarity, side ratios in right triangles are properties of angles in the triangle. Students should use this understanding to explore and understand the definitions of trigonometric ratios for acute angles. The instructional focus is for students to have opportunities to explore different right triangles that have and do not have proportional side lengths to understand that corresponding sides of right triangles with the same acute angle measure will be proportional in length.

**Level 4:**

As students solidify their understanding, they should be able to explain why different right triangles that have and do not have proportional side lengths to understand that corresponding sides of right triangles with the same acute angle measure will be proportional in length. Students should also be able to justify when right triangles will be proportional in length. Student reasoning and justifications should encompass precise mathematical vocabulary.

**Standard G.SRT.C.7 (Major Work of the Grade)**

Explain and use the relationship between the sine and cosine of complementary angles.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define sine and cosine ratios.</p> <p>Determine the measure of an angle in a right triangle by using the triangle angle sum theorem or the definition of complementary angles.</p> <p>Define complementary angles.</p>	<p>Draw a right triangle depicting given trigonometric ratios.</p> <p>Recognize complementary angles from a drawing or table.</p> <p>Recognize that the acute angles in a right triangle are complementary.</p> <p>Write a trigonometric ratio involving sine and cosine given a right triangle drawing.</p> <p>Make observations and identify patterns about trigonometric ratios from a table of values.</p>	<p>Develop logical arguments about relationships between the sine and cosine values of angles.</p> <p>Explain the relationship of the sine and cosine values of complementary angles.</p> <p>Use precise language to describe a trigonometric relationship.</p> <p>Use the concepts of the relationship between the sine and cosine values of complementary angles to solve non-routine problems such as complex drawings, embedded figures, or disseminating information that contains extraneous values.</p>	<p>Explore and draw conclusions about relationships of complementary angles using other trigonometric values.</p> <p>Critique the reasoning of others' explanation of the relationship between the sine and cosine of complementary angles.</p> <p>Create a real-world situation that would involve knowing the relationship between the sine and cosine of complementary angles.</p>

## **Instructional Focus Statements**

### **Level 3:**

Similarity, right triangles, and trigonometry are grouped together to form a domain because it is necessary trigonometry is taught through the conceptual understanding of similarity. Trigonometry is one of the most widely used mathematical concepts and serves a variety of applications in a plethora of fields, so building the foundational knowledge of the special relationships in right triangles is crucial for students to make sense of trigonometry and apply trigonometric concepts fluently. This standard expands the work students do in grades 7 and 8. In grade 7, students explore relationships in scale drawings which leads to the study of similarity. In grade 8, students explore the effects of dilations. This lays the ground work for the definition of similarity and the special case of trigonometry. With that in mind, it is important for instruction to focus on providing students with the opportunity to notice, wonder, and conjecture about patterns in a table of sine and cosine values in a right triangle. There are many patterns to explore, and instruction should promote classroom discussion to make sense of the patterns, conjecture and test hypothesis, and justify true conjectures. Hence, modeling should be a focus of instruction. Particularly, the teacher should guide the exploration to discovering the relationship between the sine and cosine values of complementary angles. Students should be required to explain why the sine of an angle is equivalent to the cosine of its complementary angle and vice versa. Once students have developed conceptual understanding of the relationship, they should be able to use this concept to solve problems.

Instruction should hold students to a high standard of using precise language and explaining their reasoning with justification. This standard is one in a cluster where many relationships of right triangles are explored such as the definition of the trigonometric ratios and the Pythagorean Theorem. Students work with three trigonometric ratios in this course: tangent, sine, and cosine. It is imperative that students develop a conceptual understanding of the trigonometric ratios and the underpinning of trigonometry. Students should begin by solidifying the procedural fluency of labeling and accurately setting up trigonometric ratios to solve applied problems.

### **Level 4:**

This standard serves as an excellent avenue for students to explore the beauty and phenomena of mathematics-particularly in trigonometric ratios. Challenge students to make sense of and conjecture about other patterns they find in a table of trigonometric values. Furthermore, instruction should provide ample opportunities for students to analyze and critique the thinking, reasoning, and arguments of others. Provide students the space to test other conjectures they find and try to prove or disprove them. Allow students the opportunity to generate and discuss real-world problems where knowing the sine and cosine values of complementary angles would be beneficial.

**Standard G.SRT.C.8 (Major Work of the Grade)**

Solve triangles.

**G.SRT.C.8a** Know and use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

**G.SRT.C.8b** Know and use the Law of Sines and Law of Cosines to solve problems in real life situations. Recognize when it is appropriate to use each.

**Scope and Clarification: (Modeling Standard)**

Ambiguous cases will not be included in assessment.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Sketch and label the sides of right triangles.</p> <p>Use trigonometric ratios and the Pythagorean Theorem to choose the unknown side lengths of a right triangle.</p>	<p>Sketch, label, and identify the trigonometric ratios of a right triangle.</p> <p>Choose which trigonometric ratio is appropriate to use in solving a right triangle.</p> <p>Use trigonometric ratios and the Pythagorean Theorem to choose the unknown side lengths and angle measurements of a right triangle.</p>	<p>Use the Pythagorean Theorem, trigonometric ratios, and the Law of Sines and Law of Cosines to solve mathematical and real life problems and recognize when it is appropriate to use each.</p>	<p>Use the Pythagorean Theorem, trigonometric ratios, and the Law of Sines and Law of Cosines to solve complex mathematical and real life problems and recognize and explain when it is appropriate to use each.</p>

**Instructional Focus Statements**

**Level 3:**

In grade 8, students develop an understanding of solving applied problems using the Pythagorean Theorem. As students encounter more complex applied problems and triangles that cannot be solved using the Pythagorean Theorem, they should develop an understanding of when it is appropriate to use the Pythagorean Theorem and when it is appropriate to use trigonometry.



Students work with three trigonometric ratios in this course: tangent, sine, and cosine. It is imperative that students develop a conceptual understanding of the trigonometric ratios and the underpinning of trigonometry. Students should begin by solidifying the procedural fluency of labeling and accurately setting up trigonometric ratios to solve applied problems.

Students should progress from using trigonometric ratios to using the Law of Sines and Law of Cosines. Students should encounter multiple examples where they develop a conceptual understanding of when it is appropriate to use the Pythagorean Theorem, right triangle trigonometry, and the Law of Sines and Law of Cosines. For any triangle, students should be able to solve for a specified angle or side and also be able to solve the triangle.

**Level 4:**

Students should solidify their procedural fluency around solving triangles using the Pythagorean Theorem, trigonometric ratios, and the Law of Sines and Cosines. They should be able to distinguish when it is appropriate to use each strategy and apply the strategy to solve applied problems. As students solidify their understanding, they should be able to explain their solution path and reasoning for their calculations, as well as their reasoning for selecting the appropriate strategy. Students should use precise mathematical vocabulary in their reasoning in written and verbal forms.

## Circles (G.C)

### Standard G.C.A.1 (Supporting Content)

Recognize that all circles are similar.

#### Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define a circle.</p> <p>Identify the center and radius of a circle.</p> <p>Use transformations to define similar figures.</p>	<p>Apply dilations to segments.</p> <p>Apply translations to points and segments.</p> <p>Recognize when translations and dilations have been applied to a circle.</p> <p>Explain the relationship between the original figure and the transformed figure.</p> <p>Understand that the length of the radius is what determines the size of the circle.</p>	<p>Recognize any two circles are similar.</p> <p>Explain in terms of transformations and by the definition of similar figures why any two circles are similar.</p> <p>Construct similar circles using dynamic geometry software or traditional tools.</p>	<p>Develop a logical argument using transformations conjecturing why any two circles can be proven similar.</p> <p>Draw conclusions about real-world problems using similarity between circles.</p> <p>Explain real-world phenomena using the relationship of similarity between circles.</p> <p>Explore and conjecture about the similarity of other curvilinear figures.</p>

## Instructional Focus Statements

### Level 3:

Students begin understanding and describing the effects of transformations on two dimensional figures in the grade 8. Foundationally, the standard 8.G.A.2 requires students to describe the effect of dilations, translations, rotations, and reflections on two dimensional figures using coordinates which prepares students to apply similarity in terms of transformations to figures. Through the study of transformations students should be able to recognize that all circles are similar. Instruction should focus on giving students the opportunity to explore similarity in terms of transformations that will relate two circles. This is an excellent opportunity to allow students time to utilize dynamic geometry software or traditional tools to construct similar circles through transformations. When circles share the same center, students can perform a dilation on the radius and draw conclusions about the resulting transformed circle. Furthermore, when circles do not share the same center, students can perform translations and dilations on one circle so that it maps onto the other and draw conclusions about their similarity. These two experiences should lead students to the understanding that all circles are similar and can be explained in terms of transformations.

Instruction of similarity in terms of transformations is much more conducive to student understanding because the similarity definition that relies on the measures of angles and sides cannot be applied to circles. While circles do not have angles and sides, students should recognize that the length of the radius is what determines the size of a circle, while they all have the same shape (by the definition of a circle).

### Level 4:

Instruction should allow students time to utilize dynamic geometry software or traditional tools to develop logical arguments about why all circles can be proven similar to one another. The use of manipulatives will help students lay the foundation for proving circles are similar. Instruction at this level should also provide students the opportunity to explore other curvilinear figures and develop conjectures about their similarity. Furthermore, instruction should provide students with circles in real-world context and ask them to make observations and draw conclusions about the modeled phenomena.

**Standard G.C.A.2 (Supporting Content)**

Identify and describe relationships among inscribed angles, radii, and chords.

**Scope and Clarifications:**

Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle, and properties of angles for a quadrilateral inscribed in a circle.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define circle.</p> <p>Define inscribed angle, circumscribed angle, and central angle</p> <p>Define and distinguish chord, diameter, and radius.</p> <p>Define tangent line and secant line.</p>	<p>Identify by correctly naming lines, chords, and angles in, on, and outside a circle.</p> <p>Draw lines, chords, and angles in, on, and outside a circle.</p> <p>Distinguish between inscribed angle, circumscribed angle, and central angle.</p> <p>Recognize the difference in a circumscribed and inscribed polygon.</p>	<p>Identify patterns and describe the relationship between a circle's arcs and angles.</p> <p>Use the relationship between arcs and angles to find unknown measures in a circle.</p> <p>Compare and contrast inscribed angles, central angles, and circumscribed angles.</p> <p>Compare and contrast secant lines and tangent lines.</p> <p>Identify, describe, and use the relationship between a radius and tangent line.</p> <p>Make observations, draw conclusions, and use the properties</p>	<p>Develop a logical argument for relationships between lines and angles found on a circle.</p> <p>Use the relationships between arcs, angles, chords, and lines found in, on, and outside a circle to solve non-routine problems.</p> <p>Apply these relationships to solve a real-world problem.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		<p>of angles in a quadrilateral when inscribed in a circle.</p> <p>Formulate conjectures and generalize findings about the relationships between angles, arcs, chords, and lines in, on, and outside a circle.</p> <p>Justify conjectures and use precise language.</p>	

### Instructional Focus Statements

**Level 3:**

This standard encompasses many relationships between angles, arcs, chords, and lines found in, on, and outside a circle. Instruction should provide students ample opportunity to explore and discover these relationships. Allow students to make observations, conjecture, and justify their conjectures. Giving students the opportunity to explore, predict, and develop their own understanding of these relationships is much more beneficial than being shown a proof of the relationship. This standard lends itself well to exploration through dynamic geometry software so students can make observations about the relationships in a fluid environment. This will allow students to conjecture, test, and develop logical arguments about the relationships. Instruction should require students to attend to precision in measuring and use precise mathematical language.

The next standard in this cluster, G.C.A.3, will require students to extend their knowledge of inscribed angles and circumscribed angles when exploring an incenter and circumcenter of a triangle.

**Level 4:**

Students at this level should focus on developing logical arguments, potentially leading into an informal proof of the relationships found in, on, and outside a circle with arcs, angles, chords, and lines. Students should be given the opportunity to use these relationships to solve for unknown values in a complex drawing where applying a variety of relationships is necessary to find all of the missing values. Furthermore, instruction should provide students problems with real-world context so that students see the benefit in knowing and being able to apply these relationships. For example, students should

use these relationships to determine the location of buildings, designing the appropriate location for appliances in a kitchen remodel, or points of interest on a city map and justify the location with mathematical reasoning.

**Standard G.C.A.3 (Supporting Content)**

Construct the incenter and circumcenter of a triangle and use their properties to solve problems in context.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define angle bisector and perpendicular bisector.</p> <p>Identify an angle bisector and perpendicular bisector in a drawing.</p> <p>Identify symbols used to indicate an angle bisector or a perpendicular bisector.</p>	<p>Bisect an angle and construct a perpendicular bisector using a compass, tracing paper, or dynamic geometry software.</p> <p>Recognize a point of intersection between angle bisectors or perpendicular bisectors.</p>	<p>Use a compass and/or dynamic geometry software to construct a triangle's incenter and circumcenter.</p> <p>Identify equal distances formed from the incenter and circumcenter.</p> <p>Know and use the properties of a triangle's incenter to find unknown measures in a figure.</p> <p>Know and use the properties of a triangle's circumcenter to find unknown measures in a figure.</p> <p>Solve real-world problems involving a triangle's incenter and circumcenter.</p>	<p>Create a real-world problem using properties of a triangle's incenter and circumcenter.</p> <p>Make observations about the relationship between an incenter and an inscribed triangle and a circumcenter and a circumscribed triangle.</p>

## **Instructional Focus Statements**

### **Level 3:**

This standard allows students to see the connection between challenging geometrical concepts and real-world context. Instruction should provide students ample opportunity to utilize their construction skills both with a compass and with dynamic geometry software in order to explore the relationship between a triangle's angle bisectors and a triangle's perpendicular bisectors. It is important to prevent students from using a ruler or protractor in these constructions so that they complete the formal construction appropriately and not by estimating the center through measurement. Students should have the opportunity to discover that the angle bisectors in any triangle intersect to form a point of concurrency called an incenter, and students should also discover that the perpendicular bisectors of any triangle intersect to form a point of concurrency called a circumcenter. It is imperative students see this relationship in a fluid, dynamic environment in order to convince themselves this relationship holds true despite how the triangle's sides and angles are manipulated. Furthermore, students should then explore and find equal distances from the point of concurrency to the sides or vertices of the triangle. Other explorations might include how the incenter divides the lengths of the angle bisector and the distance of points located on a perpendicular bisector in relation to the endpoints on the side of the triangle it is bisecting. Instruction should require students to attend to precision in measuring so that students can recognize that an incenter is equidistant to the sides of a triangle when measured perpendicularly and that a circumcenter is equidistant to the vertices of a triangle. Once students have had ample opportunity to explore and construct an incenter and a circumcenter with a variety of different types of triangles, students should then use these properties to find unknown values in a real-world context. Instruction should provide students with many applications of these special centers and require students to make decisions about appropriately locating points of interest on a map in order to meet certain guidelines. For instance, students should have to grapple with directing a community decision on where to build a park so that it is equidistant from three elementary schools. This standard is an excellent opportunity for students to apply mathematical truths in a real-world context.

### **Level 4:**

Instruction should give students the opportunity to apply the relationships of an incenter and a circumcenter to explore these mathematical truths to a real-world problem. This standard serves as a great opportunity to develop a relationship with industry or community organizations to model a problem they are potentially facing and help them make an informed decision about the location of a point of interest. For instance, an industrial company in the area may be trying to decide the best location on the premises for a water bottle refill station. Students could develop a logical argument using their knowledge of incenters and circumcenters to help the company decide the best location for the water bottle refill station so that it is convenient to many departments. Furthermore, instruction should guide students toward making connections between these points of concurrency and the center of an inscribed or circumscribed circle.



**Standard G.C.B.4 (Supporting Content)**

Know the formula and find the area of a sector of a circle in a real-world context.

**Scope and Clarifications:**

For example, use proportional relationships and angles measured in degrees or radians. There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Recognize a sector is formed by two rays of a central angle and the subtended arc.</p> <p>Calculate the area of a circle.</p> <p>Define and recognize a central angle on a circle.</p>	<p>Recognize a sector as a part of the whole measure of the area of a circle.</p> <p>Identify the measure of the degrees of a central angle of the sector as a part of the whole 360 degrees.</p> <p>Identify the measure of the radians of a central angle of the sector as a part of the whole <math>2\pi</math> radians.</p>	<p>Represent the measure of a sector as a fraction of degrees or a fraction of radians.</p> <p>Explain that the formula for area of a sector is a fraction of the circle's whole area. That is, the fraction of the circle multiplied by <math>\pi r^2</math>.</p> <p>Recognize the need for finding the area of fractional portions of a circle in a real-world context.</p> <p>Identify a sector in context of a real-world problem.</p> <p>Know and use the formula to find the area of a sector in a real-world context.</p>	<p>Create a real-world context that requires finding the area of a sector.</p> <p>Critique others' solutions and identify any errors in reasoning, misconceptions, misrepresentation of the fractional form, or errors in formula.</p> <p>Explore finding the area of a segment of a circle.</p> <p>Explore how to modify the formula for area of a sector to find arc length.</p>

## **Instructional Focus Statements**

### **Level 3:**

Instruction should guide students into recognizing that a sector is simply a fractional piece of the whole circle. Students should make use of the fact that since a sector is a fraction of the circle, then the area of a sector is a fraction of the circle's area. Instruction should encourage students to generalize the formula for area of a sector instead of memorize it. Students should see the conceptual relationship between the fractional piece of the circle and the circle's whole area. Then students can use the formula or conceptual understanding to calculate the area of sectors represented in a real-world context.

It is important to note this is the first time students will be using radians to measure angles. Instruction should provide an opportunity for students to realize radians is another way to measure angles and arcs. Radians will be developed further in Algebra II, but students need a basic understanding of them now in order to connect to the concept of this standard. Instruction should develop students' understanding that a circle has  $2\pi$  radians which is equivalent to 360 degrees. Show students what a quarter, a half, and three-quarters of  $2\pi$  is so students become more familiar with the notation of radians. Instruction should help students realize that central angles measured in radians are a portion of  $2\pi$  just like central angles measured in degrees are a portion of 360 degrees.

### **Level 4:**

Students at this level of understanding should be given the opportunity to see and hear others' solution of the area of a sector and be required to critique their reasoning and identify errors or misconceptions. Instruction should provide students with incorrect examples so students can identify errors in reasoning, misconceptions, misrepresentation of the fractional form, or errors in using the formula. This is a great opportunity to provide students the experience of this concept in a real-world setting by finding their own example of the need for the area of a sector. An excellent extension of this standard is the exploration of finding the area of a segment of a circle. Students should be allowed to discover that they will need to compute the area of the sector and then subtract the area of the included triangle in the sector. This standard also easily extends to other geometrical truths like arc length. Provide examples of arc length and ask students to generalize a formula similar to the formula for area of a sector.

## Expressing Geometric Properties with Equations (G.GPE)

### Standard G.GPE.A.1 (Supporting Content)

Know and write the equation of a circle of given center and radius using the Pythagorean Theorem.

#### Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Understand and use the Pythagorean Theorem.</p> <p>Draw a circle on a coordinate plane with given center and radius.</p>	<p>Identify a circle's center and radius when graphed on a coordinate plane.</p> <p>Describe the transformations of a circle in terms of distance of the center from the origin of a coordinate plane.</p>	<p>Write the equation of a circle centered at the origin with a given radius.</p> <p>Recognize from a graph when a circle has been translated from the origin.</p> <p>Write the equation of a circle with given center not at the origin and radius.</p> <p>Write the equation of a circle from a graph of a circle drawn on a coordinate plane.</p> <p>Make observations about the connection between the Pythagorean Theorem and the equation of a circle.</p>	<p>Make observations about the relationships between the distance formula and the Pythagorean Theorem and how they are each related to one another and the equation of a circle.</p> <p>Develop a logical argument about how the distance formula, Pythagorean Theorem, and transformations of functions are related to the equation of a circle.</p> <p>Investigate and compare the general form of a circle and the standard form of a circle.</p> <p>Use the technique of completing the square to algebraically transition into the standard form of a circle.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Explain how the equation of a circle can be derived from the Pythagorean Theorem.	

### Instructional Focus Statements

#### Level 3:

This standard ties together the work students have been doing to understand the Pythagorean Theorem and the precise definition of a circle. The cluster of standards that cover the Pythagorean Theorem are focus standards for grade 8, and students develop the precise definition of a circle in standard G.CO.A.1. It is important for instruction to make the connection between these two concepts and should first be introduced to students by examining a circle graphed on a coordinate plane and centered at the origin. Instruction should guide students through a series of questions asking students to make connections between the points on a circle and the center and grapple with how they know points lie on a circle's edge. It may be helpful for the teacher to draw in a right triangle and translate it around the circle so students can make the connection between the hypotenuse of the right triangle and the radius of the circle as well as the legs of the right triangle and the coordinates of the points that lie on the circle's edge. Hence, connecting the Pythagorean Theorem to the formal definition of a circle. This exploration should help students generalize the standard form of the equation of a circle and understand its derivation. Instruction should require students to explain the connection of the Pythagorean Theorem and the equation of a circle to peers.

After students have formulated the equation of a circle, instruction should present students with circles graphed on a coordinate plane that are not centered at the origin. Students should then explore how the translations effect the standard form of the equation of the circle. This is an excellent opportunity for instruction to connect to students' prior knowledge of transformations of functions from their study of the standard F.BF.B.2 in Algebra I. Instruction should help students see the relationship between the center of the circle (not at the origin) and the origin itself. For instance, a circle centered at (2, 1) is two horizontal units away from the origin and one vertical unit away from the origin. Instruction should connect this graphical relationship to the structure of the equation of the circle.

These connections will help students be able to explain the relationship of the Pythagorean Theorem to the standard form of the equation of a circle  $(x - h)^2 + (y - k)^2 = r^2$  and use that relationship to write the equation of a circle given center  $(h, k)$  and radius,  $r$ . This strong foundation of understanding the Pythagorean Theorem and the equation of a circle will serve useful as students explore trigonometric functions and the unit circle.

**Level 4:**

Students at this level can build on their knowledge of the equation of a circle by investigating the relationship between the Pythagorean Theorem and the distance formula. Students should be provided opportunities to explore both and make connections to how these two concepts are also related to transformations of functions. Furthermore, students should explore the general form of the equation of a circle:  $x^2 + y^2 + Cx + Dy + E = 0$  by multiplying the factors in standard form:  $(x - h)^2 + (y - k)^2 = r^2$ . Instruction should provide opportunities for students to make generalizations and draw conclusions about the relationship between the two forms. This is an excellent opportunity for students to see how their skills in completing the square from A1.A.REI.B.3.b can be used to interpret efficiently between the two equations of a circle.

**Standard G.GPE.B.2 (Major Work of the Grade)**

Use coordinates to prove simple geometric theorems algebraically.

**Scope and Clarifications:**

For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point  $(1, \sqrt{3})$  lies on the circle centered at the origin and containing the point  $(0, 2)$ . There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Know the properties of a given geometric figure.</p> <p>Recognize a geometric figure based on its properties.</p>	<p>Find measures, lengths, or distances from a given figure on a coordinate plane.</p> <p>Use algebraic notation or expressions to represent coordinates or measures on a coordinate plane.</p> <p>Know the conditions necessary for applying a geometric theorem to a figure.</p> <p>Logically organize an informal proof to justify a geometric theorem algebraically.</p> <p>Write an informal explanation of a geometric theorem.</p>	<p>Identify what measures will be needed in order to prove a geometric theorem.</p> <p>Justify properties of geometric figures algebraically using coordinates.</p> <p>Recognize when a geometric theorem is applicable to a given figure and use it appropriately in a proof.</p> <p>Prove geometric theorems algebraically using notation or expressions that represent coordinates or measures on a coordinate plane.</p>	<p>Analyze and critique proofs written by others by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument.</p> <p>Improve their own proofs based on what they have seen from others' proofs.</p> <p>Use coordinates to prove complex geometric theorems algebraically.</p>

## **Instructional Focus Statements**

### **Level 3:**

Students will likely begin this standard with the prior knowledge of using geometric theorems, but it is unlikely students have spent much time proving them. However, in grade 8 students explored the proof of the Pythagorean Theorem, so instruction should build off of that experience and extend to other geometric theorems. Instruction should provide students with the opportunity to explore geometric theorems at play on a coordinate plane. Students may struggle with representing important lengths, measures, or coordinates abstractly, so teachers should allow students the opportunity to grapple with how to represent geometric measures algebraically using coordinates. Instruction should provide students with a variety of contexts that utilize different properties of geometric figures.

This standard is an excellent opportunity to help students see the vital connection between Algebra and Geometry. Give students ample opportunity to demonstrate and justify through proof why a particular geometric theorem holds true on a coordinate plane. During instruction, students should reason quantitatively and make generalizations about coordinates, the algebraic representation of measures of a figure, and geometric theorems. Coordinate proofs should be taught simultaneously with other types of proofs such as paragraph proofs, two-column proofs, etc. This will help students see proofing has many avenues with which to accomplish it. It is imperative students be required to use precise mathematical language and make use of the structure on the coordinate plane to aide them in their proof writing.

### **Level 4:**

Instruction at this level should provide the opportunity for students to fine tune their proof writing skills by analyzing other proofs and critiquing them to find best practices in proof writing. Students should then apply what they learned through the analysis to improve their own proofs. Furthermore, students should be challenged to prove more complex geometric theorems such as Euler's quadrilateral theorem, geometric mean theorem, or Heron's formula.

**Standard G.GPE.B.3 (Major Work of the Grade)**

Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

**Scope and Clarifications:**

For example, find the equation of a line parallel or perpendicular to a given line that passes through a given point. There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define parallel lines by discussing properties of slopes and intersections.</p> <p>Identify parallel lines in a picture using symbols or given notation.</p> <p>Define perpendicular by discussing properties of slopes and intersections.</p> <p>Identify perpendicular lines in a picture using symbols or given notation.</p> <p>Recognize slope criteria for parallel lines and perpendicular lines.</p>	<p>Explain how to determine if lines are parallel or perpendicular.</p> <p>Sketch a parallel line or a perpendicular line in a drawing given a point and a line.</p> <p>Use symbolic notation to write a parallel and perpendicular statement. (e.g. <math>\overline{AB} \parallel \overline{CD}</math> or <math>\overline{PR} \perp \overline{PT}</math>)</p> <p>Informally justify why two lines in a drawing are parallel, perpendicular, or neither.</p> <p>Relate parallel lines and perpendicular lines to geometric problems.</p>	<p>Explain how the slopes of parallel lines are equivalent.</p> <p>Use the translation of the slope triangle (the right triangle formed by horizontal and vertical distances to form a triangle with the line as the hypotenuse) to justify slopes of parallel lines.</p> <p>Explain how the slopes of perpendicular lines are opposite reciprocals.</p> <p>Use the rotation of the slope triangle to justify slopes of perpendicular lines.</p> <p>Prove lines are parallel or perpendicular using slope criteria.</p> <p>Apply the properties of parallel lines</p>	<p>Explain and provide examples of the necessity of knowing whether or not lines are parallel or perpendicular in real-world context.</p> <p>Critique the work of others in using and proving slope criteria.</p> <p>Make observations and develop logical arguments about normal lines.</p>



Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		<p>and perpendicular lines to solve geometric problems.</p> <p>Write equations of parallel lines or perpendicular lines given a point and a slope.</p> <p>Write equations of parallel lines or perpendicular lines by identifying a point and a slope from a drawing.</p> <p>Use precise mathematical language and symbolic notation to describe parallel and perpendicular lines.</p>	

### Instructional Focus Statements

**Level 3:**

Understanding parallelism and perpendicularity is a vital concept to students' understanding of many different venues in mathematics. These concepts apply to geometric shapes and extend into concepts in Calculus where students will be asked to write the equation of a tangent line. Because conceptual understanding of this standard is so important, instruction should focus on allowing the students to explore and discover the relationship between slopes of parallel lines and slopes of perpendicular lines. It is beneficial to build on students' understanding of transformations, 8.G.A.1, by constructing the slope triangle for a line (i.e. a triangle that demonstrates visually the horizontal distance and the vertical distance used to create the steepness of the line) and translating the slope triangle onto the other line to determine congruence. Students can then conclude that since the slope triangles are congruent, the slope of the hypotenuse (which is a portion of the line) has the same steepness. Students should follow a similar exploration of the slope triangles of perpendicular lines where they discover the slope triangles are congruent by a 90 degree rotation. This rotation results in slopes of the hypotenuse (i.e. a segment of the line) being opposite reciprocals to one another. It is important to help students understand the reasoning behind the properties of the slopes of parallel lines and the perpendicular lines and not just memorize these properties. These explorations will prepare students to begin informally designing proofs and eventually writing their own formal proof of parallel or perpendicular lines. When writing proofs, students must use precise mathematical language and appropriate symbolic representation.

This standard is an excellent connection of Geometry and Algebra. Once students understand the reasoning behind the properties of parallel lines and perpendicular lines, they should extend their ability to write equations of lines to writing lines that are parallel to a line at a certain point or perpendicular to a line at a certain point. Instruction should provide students with problems where the student can ascertain the slope from a graph, a table, or from words. Furthermore, students should experience a variety of problems where they must apply the criteria for slope in order to solve a problem.

**Level 4:**

Instruction should provide students ample opportunity to see the necessity of parallelism and perpendicularity in real-world contexts. Students should be expected to recognize the necessity of proving lines parallel or perpendicular and be able to reason about slopes. Giving students the opportunity to hear the reasoning of how others' applied slope criteria to solve problems and analyze others' proofs can increase their level of understanding. Finally, more in depth instruction could introduce students to the concept of a normal line and present students with problems where they write the equation of a tangent line.

**Standard G.GPE.B.4 (Major Work of the Grade)**

Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify the part in a ratio.</p> <p>Identify the whole in a ratio.</p> <p>Explain the concept of part to whole in a ratio.</p> <p>Find the slope of a segment.</p>	<p>Estimate where a point would lie on a directed line segment given a ratio.</p> <p>Match ratios to partitioned directed line segments.</p> <p>Identify the direction of a line segment in terms of a given ratio.</p> <p>Determine to which endpoint the located point will be closest.</p> <p>Identify the resulting lengths of the partitioned line segment and describe their relationship to the whole segment, given a ratio.</p> <p>Partition a directed line segment on a coordinate plane.</p>	<p>Explain the importance to following the direction of the ratio when partitioning a line segment.</p> <p>Explain what it means to partition a segment into a given ratio.</p> <p>Observe patterns when subdividing line segments and draw conclusions about the effects the ratio has on the segment and its lengths.</p> <p>Use the ratio of vertical change to horizontal change (slope) to find a point that partitions a line segment into a given ratio.</p> <p>Explain the ratio of parts of a segmented line <math>a: b</math>, as <math>\frac{a}{a+b}</math></p> <p>Partition a directed line segment using a compass.</p>	<p>Make observations and draw conclusions about the cause and effect of different ratios on a directed line segment.</p> <p>Explain how to subdivide a segment given a ratio using precise mathematical vocabulary.</p> <p>Create a real-world problem where partitioning a segment appropriately is critical.</p>

## **Instructional Focus Statements**

### **Level 3:**

This standard requires students to apply their knowledge of ratios in a geometric form by partitioning a directed line segment given a ratio. Instruction should begin with having students to divide a segment on a number line and foster a discussion about what it means to a segment in a given ratio. This is a great opportunity to tie in a real-world connection like the interstate system. Students might be asked to consider a familiar section of interstate and find the midpoint of that section. Students should then be given the opportunity to explore other partitions on that section of interstate. This will help students understand the importance of direction when partitioning and how partitioning effects the lengths of the parts of the segment in regards to the length of the whole segment. This real world connection will allow students to understand the importance of attending to precision when considering the direction of partitioning and allow students to model the partitioning process using mile markers on the interstate.

Opportunities should be provided for students to investigate patterns in partitioning a segment on a coordinate plane. This investigation should begin by asking students to find the midpoint and discuss the resulting ratio of 1:1 and the lengths of the parts of the segment to the whole segment. By having students place another midpoint on one of the partitioned segments they can be guided to understanding of this new ratio of 1:3. Explore with students the lengths of the smallest part of the segment to the whole segment and ask advancing questions to help students see the relationship of a ratio  $a:b$  when comparing lengths as  $\frac{a}{a+b}$  and  $\frac{b}{a+b}$ . Instruction should also provide students with ample opportunity to explore how applying the slope to the directed line segment can partition it into a ratio. Students can apply the change in vertical distance and the change in horizontal distance repetitively in order to place the point appropriately to meet the given ratio. To aide students in their visual understanding of partitioning a segment, instruction should show students how to partition with a compass by constructing similar triangles on a line segment. Instruction should connect the numerical representation of the process of partitioning a line segment by guiding students in an exploration and having them make observations about the relationship between the x and y coordinates of the endpoints and the point of the segment's partition.

Engaging in classroom discourse will drive students to their understanding of this standard. Teachers should carefully plan their explorations and scaffold their ratios to aide in students' conceptual understanding.

### **Level 4:**

There is much essential understanding to unpack in this standard, so requiring students to explain the process of partitioning a line segment to someone with little to no prior knowledge will be a good indicator of a student's ability to extend this concept. Instruction should give students the opportunity to develop their understanding of the ratio and the resulting lengths of the segments by examining patterns and the cause and effect of a variety of ratios. Furthermore, students should recognize real-world problems that will require appropriate partitioning and explain the necessity of partitioning accurately with direction in mind.

**Standard G.GPE.B.5 (Major Work of the Grade)**

Know and use coordinates to compute perimeters of polygons and areas of triangles and rectangles.

**Scope and Clarifications: (Modeling Standard)**

For example, use the distance formula. There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify which lengths need to be used in order to compute area and perimeter of a polygon.</p> <p>Calculate the length of a horizontal or vertical segment.</p>	<p>Identify the altitude of a triangle and compute its length.</p> <p>Calculate distance between two coordinates using distance formula or the Pythagorean Theorem.</p> <p>Calculate the area of triangles graphed on a coordinate plane.</p> <p>Calculate the area of rectangles graphed on a coordinate plane.</p> <p>Use correct units when reporting answers.</p>	<p>Calculate the perimeter of polygons on a coordinate plane.</p> <p>Calculate the area of a triangle or a rectangle on a coordinate plane.</p> <p>Explain the relationship between distances or measures on a figure in terms of variables in a formula.</p> <p>Attend to precision in calculating measures.</p> <p>Justify the solution pathway for calculating area and perimeter.</p> <p>Operate with irrational numbers in radical form and write the result in simplest radical form.</p> <p>Recognize extraneous or unnecessary information.</p>	<p>Find the area of a complex figure by decomposing it into familiar shapes.</p> <p>Construct viable arguments when defending a solution pathway.</p> <p>Compare and contrast solution pathways and identify errors.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Model real-world problems by sketching them on a coordinate plane and interpret the results in context of the problem.	

### Instructional Focus Statements

#### **Level 3:**

This standard lends itself to a variety of solution pathways, so classroom instruction should provide opportunities for students to explain their thinking, justify their solution, and defend their chosen pathway. Students have prior experience with this standard from grade 6 where they had to find the area of triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes (6.G.A.1). To continue this trajectory of learning, instruction should promote student understanding through modeling and explaining real-world phenomena in terms of area and perimeter. Students should be expected to report exact answers with correct units and allowed ample opportunity to explore and discuss multiple solution pathways. Because this is a modeling standard, students should experience a variety of examples asking them to model real-world problems and represent them to scale on a coordinate plane.

Instruction should help students relate distances or lengths on a figure to the variable in the formula. For example, students may need help identifying the altitude of a figure, so it is important for students to see a variety of polygons without obvious altitudes. Explore non-routine problems and provide atypical figures that challenge students in determining the classification of the polygon. Through classroom discourse, help students recognize the base, altitude, or other important distances or lengths in formulas from a picture. Instruction should promote the use of the distance formula or the Pythagorean Theorem when computing distances or lengths, and it is imperative for students to attend to precision in their computations. In fact, students should leave their answers in simplest radical form when appropriate. This may be challenging for the students, so instruction should provide some additional support in simplifying, adding, subtracting, and multiplying numbers in radical form.

#### **Level 4:**

This standard can easily be extended by supplying students with more complex figures of which to compute perimeter and area. Allow students the freedom to explore multiple solution pathways on their own and provide for them the solution pathways of others. Ask students to find errors in the solution pathway and make suggestions for correcting it. Provide opportunities for students to construct viable arguments and defend pathways, even some they did not choose themselves.

## GEOMETRIC MEASUREMENT and DIMENSION (GMD)

### Standard G.GMD.A.1 (Supporting Work)

Give an informal argument for the formulas for the circumference of a circle and the volume and surface area of a cylinder, cone, prism, and pyramid.

#### Scope and Clarification:

Informal arguments may include but are not limited to using the dissection argument, applying Cavalieri's principle, and constructing informal limit arguments. There are no assessment limits for this standard. The entire standard is assessed in this course.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Choose and apply the formula for the circumference of a circle.</p> <p>Choose a statement that completes the argument, given a partial informal argument for the formula for the circumference of a circle.</p> <p>Apply the formula for the volume of a cylinder, cone, prism, and pyramid given the formula and a visual model.</p>	<p>Apply the formula for the circumference of a circle.</p> <p>Apply the formula for the volume of a cylinder, cone, prism, and pyramid given the formula.</p> <p>Choose a statement that completes the argument, given a partial informal argument for the formula for the volume or surface area of a cylinder, cone, prism, or pyramid.</p>	<p>Write an informal argument for the formulas for the circumference and area of a circle.</p> <p>Write an informal argument for the formulas for the volumes and surface areas of a cylinder, cone, prism, and pyramid.</p>	<p>Create a visual representation and write an informal argument for the formulas for the circumference and area of a circle.</p> <p>Create a visual representation and write an informal argument for formulas for volumes and surface areas of a cylinder, cone, prism, and pyramid.</p>

### Instructional Focus Statements

#### Level 3:

Students are applying geometric concepts learned in the middle grades in order to discuss why certain formulas work using informal arguments. The difference in this standard is that students should be able to justify why these formulas hold true.

Students began working with area in grade 3 and continued developing their conceptual understanding throughout the middle grades. This foundational understanding is the building block for students to develop a conceptual understanding of the surface area of geometric figures.

Students also began exploring circles in the early grades. The formula for the circumference of a circle can be seen when considering that, since all circles are similar, the ratios of their circumference to their diameter will be constant, defined as  $\pi$ . This is  $\frac{C}{d} = \pi$  or  $C = d \pi$ , where  $C$  is the circumference and  $d$  is the diameter of the circle.

Students should also explore the formula for the area of a circle which is foundational for understanding the volume formula for a cylinder. Students can use the Cavalier's principle to extend their observation about the area of a circle to consider the volume of a cylinder.

Students should also make the connection that the volume of a pyramid is  $\frac{1}{3}$  the volume of a rectangular prism with the same base and height resulting in  $V = \frac{1}{3}Bh$ , where  $B$  is the area of the base and  $h$  is the height of the prism. The same argument can be applied to a cone such that the volume of a cone is  $\frac{1}{3}$  of a cylinder with the same base and height resulting in the volume of a cone formula of  $V = \frac{1}{3} \pi r^2h$ , where  $r$  is the radius of the cylinder, and  $h$  is the height of the cylinder.

As students make these connections, they should be able to explain why the formulas work using drawings and models in written and verbal explanations. Central to this standard is students developing an understanding of why formulas look the way they do and an understanding of the origin of the formula as opposed to simply memorizing a formula for the sake of memorizing it.

#### **Level 4:**

As students solidify their understanding of why these formulas work using informal arguments, they should be able to explain the connections between the formulas. Students should be able to sketch drawings of geometric figures and explain the relationship between the different figures and how and why the volume formulas work. These justifications should be accompanied with the use of precise mathematical vocabulary.



**Standard G.GMD.A.2 (Supporting Work)**

Know and use volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems.

**Scope and Clarification: (Modeling Standard)**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Apply the formulas for the volume and surface area of a rectangular prism with integer dimensions given a visual model.	Apply the formulas for the volume and surface area of cylinders, cones, and spheres given the formula.	Apply volume and surface area formulas to solve mathematical and real-world problems.	Create real-world and mathematical problems and apply volume and surface area formulas to solve the problems.

**Instructional Focus Statements**

**Level 3:**

Students should use prior knowledge in order to know and use the volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems. Students should develop a conceptual understanding of applying the volume and surface area formula by making connections to the visual model and prior learnings. In standard G.GMD.A.1, students give informal arguments of volume and surface area formulas and their conceptual understandings. In this standard, students should use that developed conceptual understanding as a foundation to know the formulas and apply them to solve mathematical and real world problems.

**Level 4:**

As students solidify their understanding of surface area and volume formulas and their applications, they should begin the display procedural fluency in efficiently and accurately know formulas and using them in problems.

This standard directly addresses mathematical modeling. Given that geometric solids can be used to approximate many real life objects, the volume formulas can be used to address a broad range of contexts. Students should extend their learning to connect that not all real world scenarios can be perfectly modeled by geometric solids but it can provide approximation that yields useful information about the situation. Students should be able to explain this connection using precise mathematical vocabulary.

## MODELING with GEOMETRY (MG)

### Standard G.MG.A.1 (Major Work of the Grade)

Use geometric shapes, their measures, and their properties to describe objects.

### Scope and Clarification: (Modeling Standard)

For example, modeling a tree trunk or a human torso as a cylinder. There are no assessment limits for this standard. The entire standard is assessed in this course.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
List real-world objects that can also be described by the shape, given a geometric shape.	Choose a geometric shape that can describe an object.	Use geometric shapes, their measures, and their properties to describe objects.	Determine a geometric shape or combination of geometric shapes that can describe a real-world object.

### Instructional Focus Statements

#### Level 3:

As students solve real-world problems that involve objects, they should make the connection that geometric shapes can be used to model real-world objects. This is imperative for students to understand and apply as they solve real-world problems in future course work in G.MG.A.2. As students make this connection, they should be able to assign geometric shapes to describe objects, such as understanding that a cylinder’s measure and properties can be used to model a tree trunk or a rocket ship.

#### Level 4:

As students solidify the understanding of using geometric shapes to describe real-world objects, they should also be able to translate this understanding to using a combination of shapes to model real-world situations. As a natural integration of G.MG.A.1 and G.MG.A.2, students solidify their understanding by applying geometric methods to solve real-world problems.

**Standard G.MG.A.2 (Major Work of the Grade)**

Apply geometric methods to solve real-world problems.

**Scope and Clarification: (Modeling Standard)**

Geometric methods may include but are not limited to using geometric shapes, the probability of a shaded region, density, and design problems. There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Choose which geometric attribute(s) need(s) to be calculated in order to solve a real-world geometric problem.</p> <p>Solve mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms when a visual representation is provided.</p> <p>Solve mathematical problems involving volume of cones, cylinders, and spheres when a visual representation is provided.</p>	<p>Identify which geometric attribute(s) need(s) to be calculated in order to solve a real-world geometric problem.</p> <p>Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</p> <p>Solve real-world and mathematical problems involving surface area of cones, cylinders, and spheres.</p>	<p>Apply geometric methods to solve real-world problems.</p>	<p>Create a variety of real-world problems whose solutions require the application of geometric methods.</p>

## **Instructional Focus Statements**

### **Level 3:**

Students are applying geometric concepts learned in previous grades in order to solve real-world geometric application problems. Students should have familiarity with not only how to calculate area, volume, and surface area, but also the hallmark attributes of each.

Counter to its predecessor standards, geometric modeling is considered major work of the grade in Geometry.

Students should be formulating a strategy to solve the problem based on a mathematical understanding of the situation, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to real-world problems.

### **Level 4:**

Students should be formulating a strategy to solve the problem based on a mathematical understanding of the situation, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to real-world problems geometric problems with increased rigor over the course.